Estimation of the contribution of the effect of collisions of objects larger than 1 cm in size

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Problem

The results of studying the effect of catalogued objects' collisions on the near-Earth space (NES) contamination have been considered at last IADC sessions [1]. Collisions of smaller objects have not been considered. The author believes that this is explained by two reasons:

1. Existing fragmentation models are poorly suited to accounting for diverse collision conditions.
2. A great number (millions) of non-catalogued objects cause high methodological and computation difficulties in modeling the collisions.

That's why these problems have been primarily considered in our work.

Fragmentation model

The fragmentation model is based on the known relationship [2, 3] for a number of formed particles having mass larger than *m*

$$N(>m) = A \cdot (m/M)^B . \tag{1}$$

At catastrophic collision $M = m_1 + m_2$ is the mass of colliding objects.
Some inaccuracies of known models were revealed in the analysis process. Namely:

- The error was found in the known formula for determining the mass of a maximum fragment $m_{\max} = M \cdot (1+B)/(-B)$. The correct formula is as follows:

$$m_{\max} = M \cdot (B+1) / \left[1 + B \cdot (m_{\min} / m_{\max})^{B+1}\right]. \tag{2}$$

- There is no substantiation of choice of fragments' minimum size.
- The formula for determining the energy released at collision is a particular case of a more general formula applicable for various collision conditions

$$u = U/M = \frac{1}{2} \cdot k_1 \cdot k_2 \cdot V_{imp}^2 \tag{3}$$

Here $k_1 = m_1/M$, $k_2 = m_2/M$.

In calculating the energy required for forming the fragment having the destruction surface area S_j we use the assumption that this energy is proportional to the destruction surface area, i.e.

$$u_j = S_j \cdot g \ . \tag{4}$$

Here *g* is some constant, which depends on fragment's material.

The algorithm for calculating, on a model, the collision consequences, that takes into account the released energy estimate (*u*), consists of the following operations:

1. The maximum mass and size of fragments are calculated.
2. The cycle over possible (discrete) values of fragments' size ($d_{j=1}$), beginning with the largest fragment, is organized;
3. For each fragment size in the range of (d_j, d_{j+1}) one calculates the values of the mean destruction surface area S_j, the volume and mass value m_j. Here, the assumptions on

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[1] This work was supported by Roscosmos

possible shape and specific weight of fragments are used. The random choice of these parameters (the Monte Carlo technique) is applied;

4. The estimate (4) is calculated;
5. On the basis of model of type (1) the number N_j of fragments in the considered size range is calculated;
6. The energy $S(u_j) = u_j \cdot N_j$, required for forming all fragments of size (d_j, d_{j+1}), is calculated.
7. The estimates $Sum(u_j) = Sum(u_{j-1}) + S(u_j)$ are summarized in a cycle over decreasing size of fragments;
8. The cycle terminates when the condition $Sum(u_j) \geq u$ is satisfied.

The last size of fragments d_j is just their smallest size, which is possible for the given value of the energy (u) released at collision.

The algorithm presented above was applied for adaptive determination of parameter g which is used in formula (4) to calculate the energy expenses for fragmentation. As a result, such a value of parameter g was determined, which provides the consent of modeling results with the experimental data [4]. The results of application of adaptive updating of parameter g are presented in Figure 1. The plot shows the left boundary of fragments' size (≈ 2 mm), which corresponds to the equality of released energy (u) and energy expenses for fragmentation (the red curve). The number of fragments of size larger than 2 mm occurred to be close to 1500.

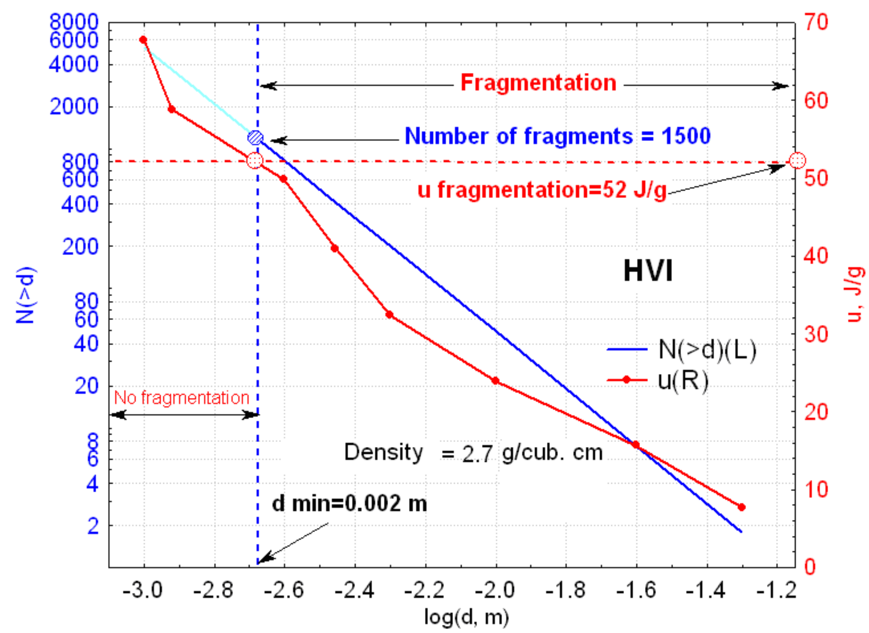

Figure 1. Modeling of high-velocity collision (HVI)

To the left side of the specified size boundary *the fragments are not formed*. The corresponding section of the curve $N(>d) = f[\log(d)]$ is painted in dark blue color. This is just that range of sizes, for which, according to the experimental data, the number of fragments does not increase with decreasing their size.

The spacecraft fragmentation model was updated on the basis of data on collision of spacecraft Cosmos 2251 and Iridium 33. In particular, the parameters of the algorithm for taking into account the variety of shapes of fragments, as well as for determining the velocity of their scattering were determined. The fraction of energy (k_v=0.1), released at collision, which is spent for increasing the velocity of fragments, was determined as

$$\Delta V = k_V \cdot u / V \,. \tag{5}$$

Making the decision, whether the given collision is catastrophic or not, is accomplished as follows. On the basis of data about impactor's and target's size one calculates the values of minimum specific energy u_1 and u_2, required for fragmentation of impactor and target, respectively. The collision, for which the conditions ($0.9 \cdot u < u_2$) and ($0.9 \cdot u > u_1$) are satisfied, is considered to be non-catastrophic. Only impactor destroys in this case.

<u>Technique</u>

The algorithm for calculating the averaged consequences of collisions consists of the following basic operations.

1) Two objects of different size are chosen.
2) The collision altitude is chosen in the altitude range from 400 eo 2000 km (with a step of 100 km), and the possible angle θ between the directions of objects' velocity vectors is chosen in the range from 0º to 360 º with a step 2 º).
3) Consequences of collisions are calculated by addressing to the standard programmed fragmentation module and are averaged with accounting for all possible basic influencing factors (random circumstances). In so doing, the following probabilities are taken into account:
   - *p(h)* – the probability of collision at the given set altitude.
   - *p(θ)* – the probability of collision at the given angle θ between velocities.
   - *p(d1)* and *p(d2)* – fractions of objects of the given size.
   - *p(d, A/m)* – the probability of the fact that the object of given size will have such ballistic factor.

The probability of collision of objects in any altitude layer is proportional to the square of density of objects in the given altitude layer. This distribution is presented in Figure 2.

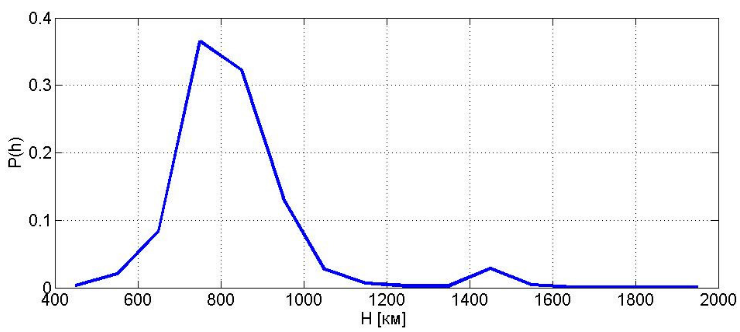

Fig. 2. Probability of collision in the altitude layer under assumption, that this probability is proportional to the square of density.

These data relate to the beginning of 2012. It is seen that the most expected region of collisions is the altitude range from 700 to 1000 km. 82 % of all collisions take place in this range.

The output data are the altitude distributions *p(d, h)* of numbers of annually formed fragments of various sizes. In this case the grid of fragments subdivision in size with the minimum fragment size of 1 mm is used.

The feature of application of the stated algorithm as compared to previous modeling of collisions consequences [1] is the essential expansion of the range of sizes of colliding objects. In this connection, as well as for convenience of comparison with the previous results, all possible collisions are subdivided into 3 types (groups):

Group 1. Mutual collisions of space objects (SOs) in the size range from 1 cm to 20 cm.

Group 2. Mutual collisions of catalogued SOs larger than 20 cm in size.

Group 3. Collisions of SOs in the size range from 1 cm to 20 cm with catalogued SOs larger than 20 cm in size.

As a result of modeling the consequences of collisions for all three mentioned types, the probabilities of possible collisions in each group were determined. They are presented in Table 1.

Table 1. Probabilities of collisions of various types

Group No, *i*	1	2	3
Probability P_i	0.967272	0.000269	0.032259

The sum of these probabilities is equal to 1.0. The important feature of these estimates is the fact that *the probability of mutual collisions of non-catalogued SOs of size from 1 to 20 cm is ≈3600 times greater than the probabilities of mutual collisions of catalogued SOs*.

In the course of application of the above algorithm the results have been attributed to some particular group. Three $p(d,h)_i$ distributions were constructed attributed to these groups. These distributions were obtained under the condition, that one collision of SOs larger than 1 cm in size took place.

As an example, Figure 3 presents the *p(d,h)* distribution for the consequences of collisions of 1-st type.

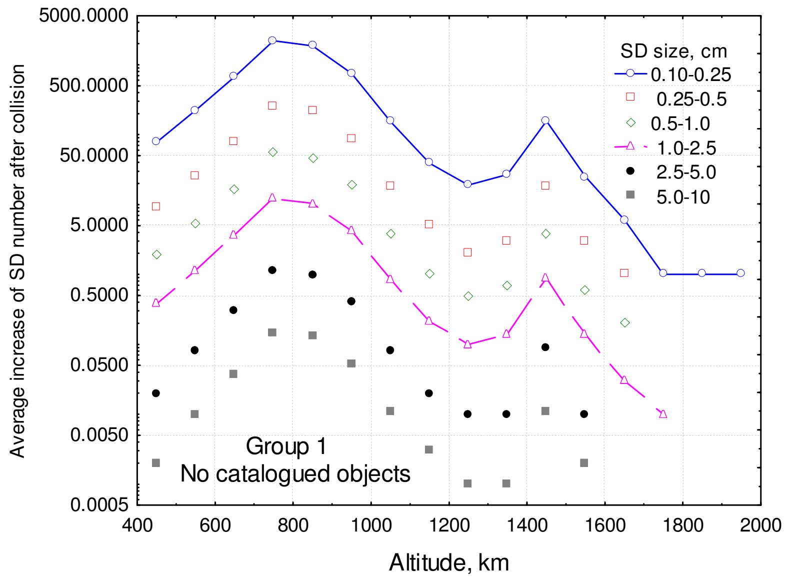

Fig. 3. Increment of a number of objects at SO collisions from the 1-st group (2012)

The tests of fragmentation model were carried out by comparing calculation results obtained with using the new model and the model developed in 2010. The estimates obtained by 2010 and 2012 models were found to be differed in the minimum fragment size only. According to the 2012 model estimates, the minimum size is 100 times greater. This is due to accounting the energy expenses for destruction. The same difference takes place at comparing the modeling results with NASA's test example [5] for catastrophic collisions.

Modeling results

Let the probability of catalogued SOs collisions per time unit be equal to some value P_{cat}. We will compare, for example, the number of fragments of size 0.10–0.25 cm, which are formed at the altitude of 700–800 km, provided that one collision of catalogued SOs took place during the time interval $1/P_{cat}$. On the same interval the expected number of collisions of 1st type will be P_1 / P_2 = 3600 times greater, and the similar number of collisions of 3rd type will be P_3/P_2=120 times greater. Corresponding estimates are presented in Table 2.

Table 2. Number of fragments of size 0.10–0.25 cm at the altitude of 700–800 km, which are formed on the interval $1/P_{cat}$ (for one collision of catalogued SOs)

Type of collisions i	1	2	3
Number of fragments N_i	$7.90 \cdot 10^6$	$0.87 \cdot 10^6$	$1.39 \cdot 10^6$
Ratio N_i / N_2	9.1	1.0	1.6

It is seen from these data that the "contribution" of collisions of 1st and 3rd types into formation of small-size fragments is much higher, than the "contribution" of collisions of 2nd type (9.1 times and by 60 %, respectively).

The technique of calculating the probability of collisions P_{cat} consists is as follows [6, 7]. The mean number of collisions N_{col} of objects of various sizes per time unit is equal to the product

$$N_{col} = \overline{\overline{F}}_{Dd} \cdot \left[0.5 \cdot \sum_j n(h_j, h_j + \Delta h) \cdot \overline{Q}(h_j) \right] \quad . \tag{6}$$

Here factor 0.5 is applied in order that the mutual collisions be not taken into account twice. For small values $N_{col} \ll 1$ this quantity can be treated as the probability of collisions. In the general case this quantity has a meaning of mean number of collisions per time unit.

For catalogued SOs the quantities sizes used in formula (6) are determined as follows.

$\overline{\overline{F}}_{Dd}$ is the estimate of the mean area of two colliding catalogued SOs. According to the data of monograph [1], for catalogued SOs $\overline{\overline{F}}_{Dd}$ = 3.49 sq. m.

$n(h, h + \Delta h)$ is the number of catalogued objects in the range of altitudes (h, $h+\Delta h$).

$\overline{Q}(h_j) = \overline{\rho}(h_j) \cdot \overline{V}_{rel}(h_j)$ is the mean value of flux density at the given altitude, which is equal to the product of the mean density by the mean value of relative velocity.

As a result, according to the catalogue data, for the beginning of 2012 (13100 objects) the following estimate was obtained:

$$P_{cat} = N_{col} = 0.218 \text{ collisions per year.} \tag{7}$$

This probability (the mean number of collisions) changes in the course of situation forecasting. For calculating this quantity at the arbitrary time instant it is convenient to make use of the approximate relation

$$N_{col}(t) = N_{col}(t_0) \cdot \frac{\left[\sum_j n(h_j, h_j + \Delta h) \cdot \overline{Q}(h_j) \right](t)}{\left[\sum_j n(h_j, h_j + \Delta h) \cdot \overline{Q}(h_j) \right](t_0)} \approx N_{col}(t_0) \cdot \left[\frac{\rho(h,t)}{\rho(h,t_0)} \right]^2 \quad . \tag{8}$$

Here one should choose the altitude to be such one, where the density is maximum. According to the data of Figure 2, this altitude lies in the range from 700 to 900 km.

Now we consider, how the probability of collisions of non-catalogued SOs is determined. In this case it is expedient to make use of the probabilities of collisions of SOs of various types (from various groups), presented in Table 1 (which were obtained under the condition of one collision of SOs larger than 1 cm in size) and the estimate (7) of probability of collision of catalogued SOs. It follows from this consideration, that the estimates of the mean number of collisions of objects of 1st and 3rd types per year at the initial instant t_0 are equal to:

$$N_{col}^{(1)}(t_0) = P_{cat} \cdot \frac{P_1}{P_2} = 783, \tag{8}$$

$$N_{col}^{(3)}(t_0) = P_{cat} \cdot \frac{P_3}{P_2} = 26. \tag{9}$$

To estimate the contribution of consequences of collisions into the level of NES contamination by small-size fragments, the situation forecasting since 1990 to 2010 was performed.

Table 3 presents the estimates of a number of objects in the altitude range from 400 to 2000 km at the initial time instant, as well as in 2010.

Table 3. Number of objects of various sizes

Year	Range number and corresponding SO sizes, cm							
	1	2	3	4	5	6	7	8
	0.1-0.25	0.25-0.5	0.5-1.0	1.0-2.5	2.5-5.0	5.0-10	10-20	>20
Model, 1990	$76.75 \cdot 10^6$	$5.66 \cdot 10^6$	975000	144500	28000	8960	2830	5320
Forecast, 2010	**$597.0 \cdot 10^6$**	**$67.7 \cdot 10^6$**	**$14.6 \cdot 10^6$**	**$3.3 \cdot 10^6$**	**352000**	**59800**	6350	5110
Model, 2010	$193.6 \cdot 10^6$	$14.5 \cdot 10^6$	2.45·106	367000	71150	22600	7060	13120
Ratio	3.1	4.7	5.9	9.0	5.1	2.6	0.90	0.39

The forecast was carried out under assumption, that in the time interval since 1990 to 2010 the only source of space debris formation were the cases of SO fragmentation as a result of mutual collisions. The second line of the table presents the results of situation forecasting since 1990 to 2010 taking into account mutual collisions of objects larger than 1 cm in size, the third line – corresponding data of the SDPA model for 2010, and the last line – the ratio of forecasted and modeled estimates for 2010.

The obtained results are rather unexpected. They testify to a very strong influence of mutual collisions on the NES contamination by fragments of size from 1 mm to 5.0 cm. As compared to SDPA model estimates for 2010, the forecasted number of fragments of given size occurred to be 5 – 9 times greater.

For objects of size from 10 to 20 cm the forecasted and modeled estimates agree well enough, and for catalogued SOs (larger than 20 cm in size) the forecasted estimates occurred to be 2.5 times lower. This result is a natural consequence of low probability of mutual collisions of catalogued SOs, as well as a consequence of great contribution of launches of new spacecrafts and their unforeseen destructions, which were disregarded in situation forecasting.

Table 4 presents the estimates of the mean number of mutual collisions over the forecasting interval. The estimates relate to three types of collisions considered above.

Table 4. Mean number of mutual collisions over the forecasting interval

Group number	1	2	3
Number of collisions	2330	0.65	78

Remind that the 1st group includes mutual collisions of objects from 1 cm to 20 cm in size. The expected number of such collisions exceeded 2000 over the time interval since 1990 to 2010.

To clearly explain the contribution of mutual collisions Figures 4 and 5 present altitude distributions of fragments of size from 1 to 2.5 mm and from 1 to 2.5 cm, which correspond to the versions considered above.

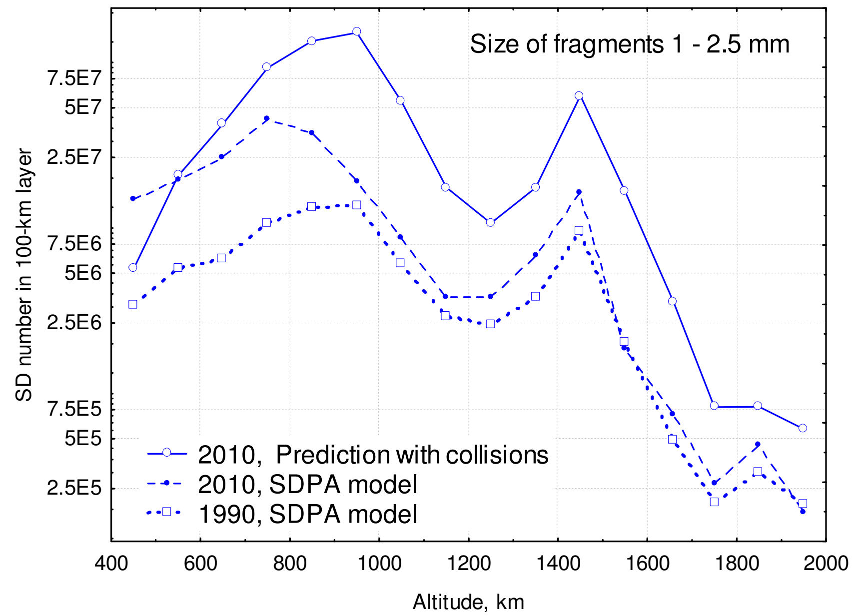

Fig. 4. Comparison of altitude distributions of fragments of size 1.0 – 2.5 mm

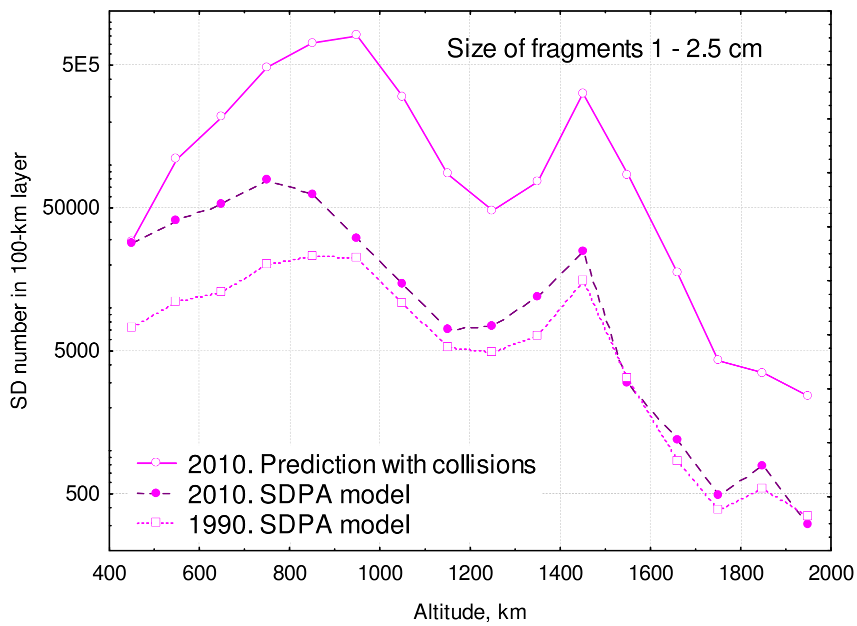

Fig. 5. Comparison of altitude distributions of fragments of size 1.0 – 2.5 cm

It is seen from these data that the greatest distinctions of forecasting results, obtained with accounting for collisions, from the modeling data take place in the altitude range from 700 to 1100 km, where the estimates of a number of fragments differ 10 times. This is a natural consequence of maximum density of SOs at these altitudes. Besides, one can see weakening of the atmospheric drag effect with growing altitude. At low altitudes the fragments are highly decelerated and quickly burn down. At high altitudes this process proceeds much more slowly. By this reason the accumulation of fragments is observed there.

Comment. The data on the altitude distribution of collision fragments, namely, their great quantity at altitudes higher than 700 km does not agree with the modeling data. The possible explanation of this fact consists in the practical absence of measurements of small-size fragments of space debris at these altitudes. So, this result was not reflected in the models. Besides, in all space debris models the given source of formation of small-size fragments was not considered. The materials of this report are, apparently, first estimations of the contribution of consequences of mutual collisions of objects of various sizes into the NES contamination by fragments larger than 1 mm in size.

Figure 6 presents the typical example of change of a number of small-size fragments of collisions on the forecasting interval, namely, the number of formed fragments, which burned down and remained in the NES.

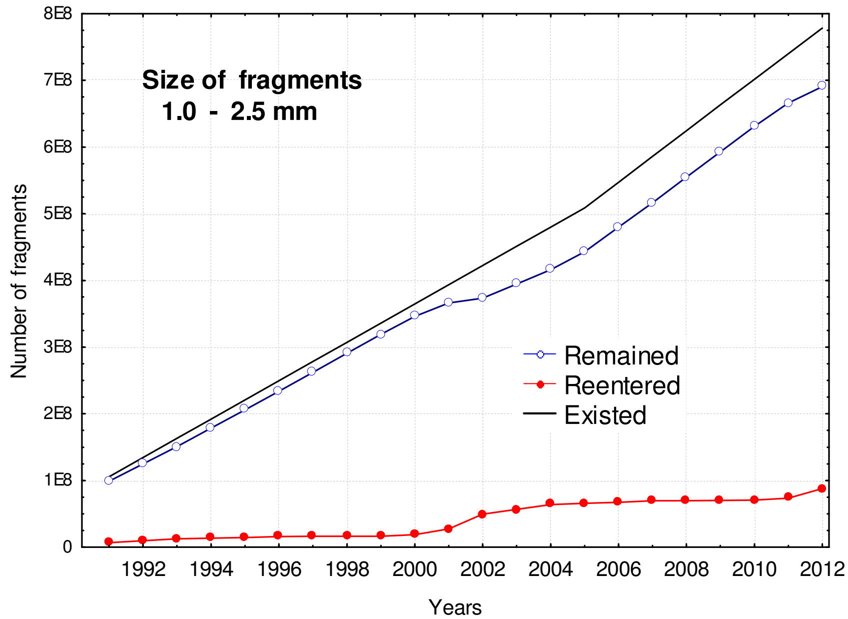

Figure 6. Change of a number of fragments of size 1-2 mm on the forecasting interval

It is seen from these data that ≈30 000000 fragments of size from 1.0 to 2.5 mm have been annually formed on the forecasting interval. About 10 % of them burned down as a result of atmospheric drag effect. Variations of a number of burned-down objects are explained by the change of drag intensity in connection with the 11-year solar activity cycle.

It is useful to compare the obtained results with the data of the known Figure 7 presented below. This figure shows the generalized estimates of a flux of SOs of various size constructed on the basis of space debris measurements [8].

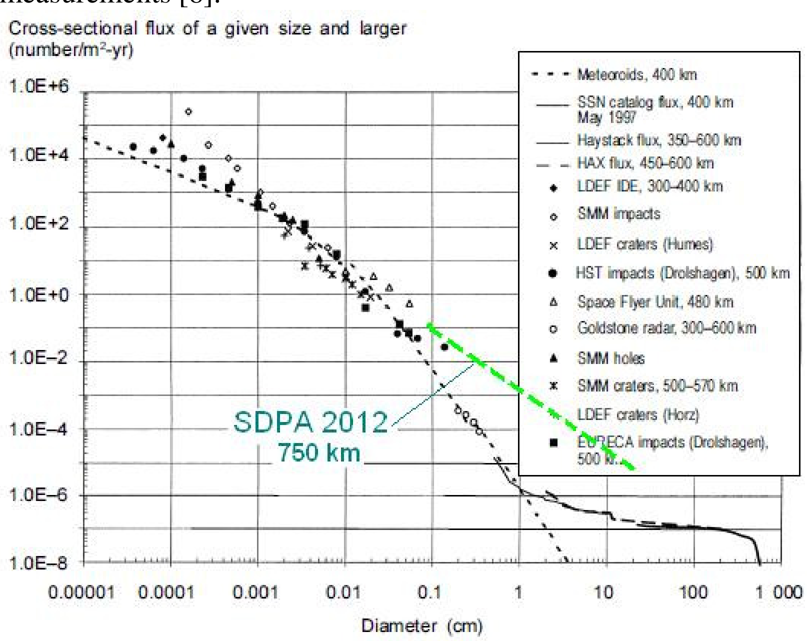

Figure 7. Characteristics of space debris flux according to the data of various sources and the SDPA estimates with accounting for mutual collisions

Here the generalized data on space debris measurements are presented. The regions, where the measurements have been carried out, are shown. A prominent feature of these data is monotonous growth of a number of objects with decreasing their size. In addition, the green dashed line shows the obtained dependence. The maximum distinction of these estimates from the radar measurement data is 100-fold. In estimating this fact one should have in mind that the measurement data were obtained at rather low altitudes and, also, that they have been obtained till 2000. So, the consent of obtained estimates with the generalized data on flux measurements should be considered acceptable.

Thus, the basic source of formation of small fragments are mutual collisions of non-catalogued objects. The contribution of this source into forming space debris of size from 1 mm to 10 mm is an order of magnitude greater than the contribution of mutual collisions of catalogued SOs.

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