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STATE TRANSITION MATRIX OF RELATIVE MOTION FOR THE NONCIRCULAR ORBIT. RELATION WITH PARTIAL-DERIVATIVE MATRIX IN THE SATELLITE COORDINATE SYSTEM

Andrey I. NAZARENKO
Space Observation Center, Russia
nazarenko@iki.rssi.ru

ABSTRACT

The analytical formulas for the partial-derivative matrix $U(t,t_0)_{RSW}$ in the satellite coordinate system were derived by the Russian experts P.E. Elyasberg [1] and V.I. Charniy [2] at the beginning of sixties. This matrix includes the effect due to the reference orbit eccentricity. The relation between state vector variations at some time instant in geocentric celestial (*x. y, z*) and satellite coordinate system (*R, S, W*) is appeared by the orthogonal transformation matrix *G*

$$\left\| \frac{\partial(x,y,z,v_x,v_y,v_z)}{\partial(R,S,W,V_R,V_S,V_W)} \right\| = \left\| \begin{matrix} G & 0 \\ 0 & G \end{matrix} \right\|.$$

It is shown in this paper that the mentioned matrix $U(t,t_0)_{RSW}$ can be serve as the basis for calculating the state transition matrix of relative motion for a non-circular orbit. The derived formulas

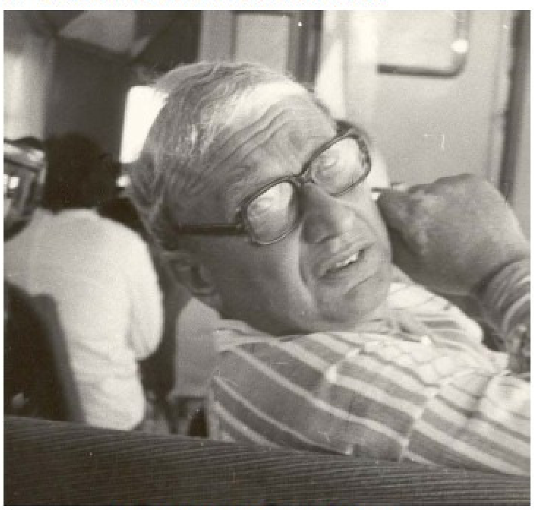
$$U(t,t_0)_{rel} = L(t)\cdot U(t,t_0)_{RSW}\cdot L(t_0)^{-1},$$

$$L(t) = \left\| \begin{matrix} E & 0 \\ -G^T\dot{G} & E \end{matrix} \right\|,$$

being rather simple in appearance, represent generalization of the well known solution of the Clohessy-Wiltshire equations, suitable for circular orbits only.

This paper presents as well the results of the matrix $U(t,t_0)_{rel}$ calculation by using the developed method and their comparison with results of numerical integration and application of the solution of the Clohessy-Wiltshire equations for circular orbits.

1. RESULTS OF P.E. ELYASBERG



Pavel E. Elyasberg, 1914-1988

Apparently, in the Russian literature, the case under consideration was stated for the first time in the monograph by P.E. Elyasberg [1] and in the publication by V.I. Charniy [2]. The solution of the problem is presented in P.E. Elyasberg's monograph (Section 10.11 «Partial-derivative matrix of the current characteristics under the initial motion data in the rectangular coordinate system (at t=const)») in the most complete form.

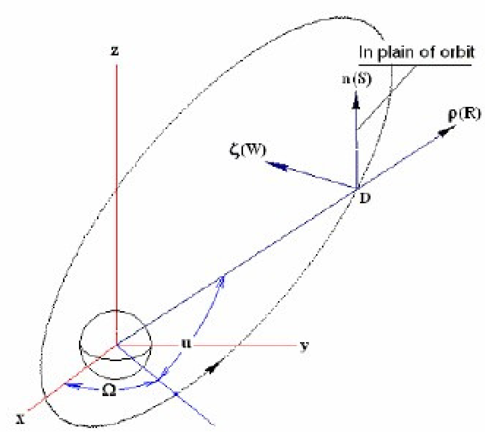


Figure 1. The orbital coordinate system

The technique and results of the partial-derivative matrix construction in the orbital coordinate system