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ERRORS OF FORECASTING OF SATELLITE MOTION IN THE EARTH GRAVITY FIELD

Obtaining of authentic statistical estimates of errors is topical for solution of many application problems, for example, for estimating the probability of collisions of satellites, for navigation with using dedicated satellite systems, etc. The content of the book is the development of the technique for estimating satellites' motion errors caused by inaccuracy of the known parameters of Earth's gravity field. Four basic sections of the book outline the following issues: 1) principles of the technique for forecasting correlation matrixes of errors in a satellite's state vector; 2) statistical characteristics of gravitational disturbances; 3) forecasting of the correlation matrix of errors; 4) adjustment of the algorithm parameters.

The book presents numerous examples of solution of considered problems with using the model and real information. The texts of four applied computer programs are given in the Appendix, namely: 1) the program for constructing the autocorrelation functions of random gravitational disturbances in the Earth-centered coordinate system; 2) the program for forecasting the correlation matrix of state vector errors with accounting for a «pure» effect of gravitational disturbances; 3) the program for analyzing the accuracy of so-called TLE; 4) the program for determining the RMS of measurement errors and system's noise using the simulation model.

The author hopes that materials of the book will be useful for a rather wide scope of specialists: researchers, engineers, post-graduate students and students. The list of used sources includes 44 names.

Keywords: gravitational field of the Earth, statistical characteristics, gravitational disturbances, correlation matrix, correlation function, forecasting of errors, adjustment of parameters, computer programs.

Получение достоверных статистических оценок погрешностей является актуальным для решения многих прикладных задач, например, для оценки вероятности столкновений спутников, для навигации с использованием соответствующих спутниковых систем и др. Содержание книги — развитие методики для оценки погрешностей прогнозирования движения спутников, порождаемых неточностью известных параметров гравитационного поля Земли. В четырех основных разделах книги изложены следующие вопросы: 1) основы методики для прогнозирования корреляционных матриц погрешностей вектора состояния спутников; 2) статистические характеристики гравитационных возмущений; 3) прогнозирование корреляционной матрицы погрешностей; 4) настройка параметров алгоритма.

В книге приводятся многочисленные примеры решения рассматриваемых задач с использованием модельной и реальной информации. В приложениях представлены тексты четырех прикладных программ: 1) построения автокорреляционных функций случайных гравитационных возмущений в связанной с Землей системе координат; 2) прогнозирования корреляционной матрицы погрешностей вектора состояния с учетом «чистого» влияния гравитационных возмущений; 3) анализа точности так называемой TLE; 4) определения СКО погрешностей измерений и шума системы на имитационной модели.

Автор надеется, что материалы книги будут полезны для достаточно широкого круга специалистов: научных работников, инженеров, аспирантов и студентов. Список использованных источников состоит из 44 наименований.

Ключевые слова: гравитационное поле Земли, статистические характеристики, гравитационные возмущения, корреляционная матрица, корреляционная функция, прогнозирование погрешностей, настройка параметров, компьютерные программы.

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ПРИБОРОСТРОЕНИЯ И ИНФОРМАЦИОННЫХ СИСТЕМ»

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**ПОГРЕШНОСТИ ПРОГНОЗИРОВАНИЯ
ДВИЖЕНИЯ СПУТНИКОВ
В ГРАВИТАЦИОННОМ ПОЛЕ ЗЕМЛИ**

ПОД РЕДАКЦИЕЙ
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СЕРИЯ
МЕХАНИКА, УПРАВЛЕНИЕ И ИНФОРМАТИКА

Москва, 2010

**ERRORS OF SATELLITE MOTION FORECASTING IN THE
EARTH GRAVITY FIELD**

Introduction

The traditional approach to forecasting celestial bodies' motion, advanced in classical works on celestial mechanics, is based on deriving and solving the equations of motion. This approach is characterized by application of the assumption, that: 1) all forces, acting on a space object, are known and can be taken into account in the right-hand parts of the differential equations of motion, and 2) the basic source of prediction errors are the errors in the initial conditions. The latter ones arise at determination of orbital parameters (the initial conditions) by the least squares techniques (LST), because the measurements contain the errors. The traditional approach is widely used for SO's motion prediction. However, it occurs that not all forces acting on an artificial Earth satellite can be considered to be known and that, in many cases, not all of known disturbing forces can be taken into account in the right-hand parts of the differential equations of motion in the process of their integration. The errors in the model of motion, arising in this connection, can have negative effect on the results of SO's orbit determination and prediction. For example, one of negative influences of these errors is restricting the time interval (fit span), on which the measurements are used for determining the initial orbital conditions.

The new field of classical celestial mechanics application arose after launching the first artificial satellite of the Earth in the Soviet Union in 1957 and the deployment of a wide range of space researches. As a result of a great number of studies on this subject, the new section of science – space ballistics (Astrodynamics) was formed. Its content was essentially influenced by a number of circumstances:

- Launching of a great number of Artificial Earth Satellites (AES) solving various application tasks (more than 600 active spacecraft are flying currently in the near-Earth space (NES)).
- Forming of a great number of "background" space objects. The Russian and American Space Surveillance Systems (SSS) are performing their detection and monitoring of the current state. Nowadays, more than 13000 satellites larger than 10-20 cm in size are contained in the operatively updated Catalogue of these systems. This space debris became a “scary headache” for the experts engaged in solving various application tasks in NES.
- A real danger of accidental collisions of satellites arose. The latest example is the collision of the American active spacecraft (SC) “Iridium 33” with the Russian disabled SC “Kosmos 2251” on February 10, 2009. As a result, more than 700 catalogued objects were formed.

- The development of a stochastic approach to describing the motion of satellites, where the random disturbances are taken into account in the right-hand parts of the equations of motion.
- The availability of orbital data on a large number of satellites (for example, <http://www.space-track.org>) and of a number of computer application programs. You can "download" them from corresponding sites in the Internet.
- Phenomenal advances in the computer technology.

A characteristic feature of space ballistic implementation is massive calculations. Hundreds of thousands of satellites motion forecasts are running daily in the world. Of course, under these circumstances the necessity and possibility of having statistical estimates of motion forecasting errors arose.

Making of robust statistical estimates is especially relevant to solving many application tasks, such as estimating the probability of collisions, providing navigation with using satellite systems, and others. The a posteriori analysis of forecasting errors is widely used in solving this task. Such analysis is quite relevant for a limited number of specific satellites. However, because of a large diversity of orbits, this approach is not always possible. The development of the methodology for obtaining a priori statistical characteristics of satellite motion prediction errors, suitable for various types of orbits, is one of new tasks of space ballistic.

The early author's studies on the development of a statistical approach to a priori estimation of motion prediction errors date back to the end of sixties. They are characterized by applying the "color" noise to accounting for random perturbations. This approach was pioneer one for that time. The results were published in a number of articles [1, 2] and monograph [3]. The author thanks M.D. Kislik, Yu.P. Gorohov, A.D. Kurlanov, V.I. Mudrov, R.R. Nazirov, L.P. Pellinen and P.E. Elyasberg for supporting these studies and participating in discussion of the results. In addition, he feels necessary to note an important contribution in the practical implementation of relevant algorithms made by his colleagues: V.D. Anisimov, A.G. Klimenko, I.M. Kutepov, L.G. Markova and I.G. Pozdnyakov.

40 years have passed since the aforementioned studies. Great changes took place in the considered area during this time, namely:

- the great progress is achieved in updating the parameters of the Earth gravitational field ;
- the construction of a priori statistical characteristics of motion errors became a more urgent problem;
- several works on this issue were published in the American editions;
- computer technology characteristics have improved by several orders of magnitude.

In the light of the changed situation, the necessity arose to improve the earlier developed approach to calculating the a priori statistical characteristics of motion

errors with using Earth's gravity field and, on this basis, to make accessible the relevant algorithms and software for consumers. The author thanks K.T. Alfriend, Professor of the Texas A&M University, for the interest and supporting further studies on this issue¹.

The book is intended mainly for the experts solving application tasks in the field of space ballistic. In substantiating the relevant algorithms, the author tried not to go deep into mathematical thickets, which are characteristic for solution of tasks with regard to stochastic factors. The book contains numerous examples of solving considered tasks with using the model and real information. So, the author hopes that professionals with engineering training will be able to understand the material of the book.

The book focused on the issues, which are insufficiently elucidated in scientific publications. In this case, the author tried to present the rationale of algorithms fully enough. Although it may be difficult for professionals with engineering training to assimilate this material, nevertheless, it seems necessary for them. When using known methodological techniques, we give references to the relevant publications.

The texts of the main software in source's codes are presented in the book to facilitate implementation of developed techniques and algorithms for solving application tasks. This software can be useful not only for engineers-researchers, but for students and graduate students as well.

¹ A.I. Nazarenko, K.T. Alfriend. Development of the technique for the covariance prediction using the gravity color noise. AAS/AIAA 2009 Space Flight Mechanics Meeting, Savannah, GA, February 8-12, 2009, AAS 09-230.

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Conclusions

The characteristic feature of the approach to solution of the problems, considered in the monograph, lies the fact, that gravity disturbing accelerations, disregarded in the models of motion, are assumed to represent the stationary Gaussian color noise. Methodological issues, related to construction of statistical characteristics of these disturbances and their application to forecasting the correlation matrix of state vector errors, are presented in detail in sections 2 and 3.

The approach, stated in the monograph, differs from traditional one. As usual, the applied ballistic satellite tasks are solved by applying the Least Squares Technique (LST), in which the weighting matrix does not take into consideration the effect of random disturbances. So, the correlation matrix of LST values is inhibitory to obtain the reliable data concerning forecasting errors.

It was shown in section 1 that, in satellite motion forecasting, the mutual correlation arises between state vector errors and noise at various time instants. So, this mutual correlation should be taken into account to ensure correct forecasting the correlation matrix of state vector errors. Forecasting without such a mutual correlation leads to incorrect results.

The important applied recommendation follows from the materials of sections 1 and 4. Namely, to correctly determine statistical characteristics of forecasting errors it is necessary to pass from LST to the modified maximum likelihood technique (MLT), in which the weighting matrix takes into account the influence of random disturbances. The effect of applying this technique consists not only in more reliable estimates of errors, but also in improving the motion prediction accuracy. The last circumstance is urgent for many applications.

Under real conditions, along with gravitational disturbances, some other factors influence the satellite motion, the atmospheric disturbances in particular. It was noted in section 4, that significant work has been done in recent years to identify atmospheric density variations. As a result, essential progress was achieved in developing the models of motion and in studying random errors of motion forecasting caused by atmospheric drag fluctuations. The detailed presentation of relevant methodological issues and investigation results is beyond the task of this monograph. Nevertheless, the materials of sections 1 and 4 are applicable not only to gravitational disturbances, but to disturbances of other origin as well.

It was noted in the introduction, that the development of methodology for obtaining a priori statistical characteristics of satellite motion prediction errors, suitable for various types of orbits, is one of new tasks of space ballistics. Obtaining of robust statistical estimates is especially topical for solving many application tasks, such as:

- choosing the fuel reserve for maintaining specified orbital parameters;
- controlling the motion of a satellite during its lifetime;
- correct use of the functional satellite information obtained during the satellite operation;
- prediction of time and place of satellite reentry;
- evaluation of the probability of satellite collision with other objects and taking measures for preventing any collision.

Reliable estimation of forecasting errors is also relevant for monitoring the flight of many disabled objects (space debris). Space debris monitoring is carried out by the space surveillance systems, which perform cataloguing of all accessible objects. Because of a great number of space debris objects, the reliable identification of obtained measurement data with catalogued objects and detection of new objects are very difficult problems. The reliable estimation of forecasting errors plays decisive part here.

In connection with the growth of manmade space debris, the probability of SC collisions with smaller objects, not susceptible for cataloguing yet, has increased. The problem of decreasing the

lower boundary of catalogued objects' size is very topical now. Solution of this problem on the basis of attracting additional measurement means requires not only insurance of reliability of statistical estimates of errors, but also improvement of the accuracy of determination and prediction of orbits.

Improvement of applied computer programs, in which the estimation of satellite forecasting accuracy plays important part, is a rather complicated problem. Along with overcoming objective difficulties, one should have in mind the subjective factor as well. Generally, specialists are focused on application of the traditional approach; they treat the innovations rather skeptically. Nevertheless, in author's opinion, the objective reality lies in the fact, that there is no alternative to the approach stated in the monograph. Sooner or later, this approach will take a worthy place in applications. The author hopes that software described in annexes would facilitate the introduction of algorithms, presented in the monograph, into the practice of applied ballistic calculations.

Appeal to readers:

The author will be much obliged to everybody, who will send him the comments and suggestions.

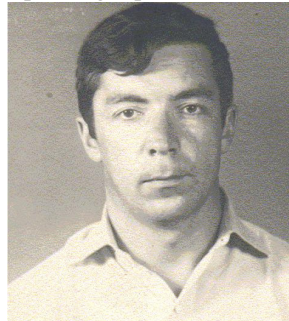
Appendix G

Fragments of the monograph history

G.1. The author's history in photographs



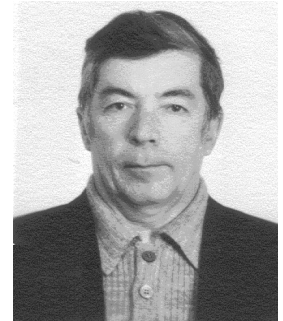
1963



1965



1971



1990



1991



1995



1997



2002

G.2. Episodes

Scene 1 (in late sixties). Head of the Institute and Project Manager.

Project Manager: "Our staff, I.I. Ivanov, P.P. Petrov and S.S. Sidorov, have developed the new model of motion for satellites. Its name is: Standard Block for Prediction (SBP). The algorithm and software are adjusted, tested and successfully worked in the automated information processing system. The accuracy and speed characteristics well satisfy the relevant requirements".

Head of the Institute: "Very good. Prepare the proposals for encouraging the staff".

Scene 2 (some years later). Head of the Institute and Project Manager.

Project Manager: "Our staff, I.I. Ivanov, P.P. Petrov and S.S. Sidorov, have developed the new algorithm and software for prediction of a correlation matrix of errors. Its name is: Standard Block of Errors (SBE). The algorithm and software are adjusted, tested and successfully worked in the automated information processing system. The results well satisfy the relevant requirements, namely, the correspondence between real and a priori statistical characteristics of forecasting errors is achieved.

Head of the Institute: "Very good. Prepare the proposals for encouraging the staff. But I want to note that the software name is not very good. I propose a more appropriate name: Universal Effective Block of Errors (UEBE)".