ISSN 0038-0946, Solar System Research, 2013, Vol. 47, No. 7, pp. 564–568. © Pleiades Publishing, Inc., 2013.
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Application of the Method for Optimum Filtering of Measurements for Determination and Prediction of Spacecraft Orbits

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Received February 24, 2012

Abstract—The article covers the outcome of the method for optimal filtering of measurements developed by the author and aimed at the determination of the time and place of reentry of the *Phobos-Grunt* spacecraft. So-called two-line elements (TLE) of the orbit of the American Space Surveillance System are used as measurements.

Keywords: orbit determination, motion prediction, *Phobos-Grunt*, atmospheric reentry
DOI: 10.1134/S0038094613070113

INTRODUCTION

During the first years of space exploration, a need arose for the determination of lifetimes of satellites before launch during the design stage (King-Hele, 1964). Later, this problem attracted the attention of specialists after the *reentry of several dangerous large satellites, such as SkyLab, Kosmos-954, Kosmos-1402, Salyut-7/Kosmos-1686*, etc. (Re-entry of space debris, 1985; The reentry of Salyut-7/Kosmos-1686, 1991).

Most dangerous falls were characterized by the absence of communications with the satellites and possibilities for their control. The only sources of initial orbital data for solution of the above problem in this case are the Russian and American Space Surveillance Systems (SSS). These systems were developed in the interests of the corresponding military services; therefore, access to the SSS data is limited. Nevertheless, orbital data on many satellites in the form of so-called two-line elements (TLE) are routinely updated online on the site of the American SSS (http://www.space-track.org, 2012).

The technique for solution of the problem under consideration is based on integration of the equations of motion under known initial conditions, which consist of a six-dimensional status vector and estimation of the braking parameter. Different characteristics are used as the braking parameter. Estimates of the ballistic coefficient (Sb) and variations in the period per revolution under atmospheric effects (ΔT) are the most popular.

A feature of the solution of the problem is the sensitivity of the results to the accuracy of the initial braking parameter. The point is that the lifetime of satellites is inversely proportional to the braking parameter $t_{\text{life}} \approx C/Sb$, where C is a constant (Nazarenko and Skrebushevskii, 1981). This implies an important dependence for the estimation of lifetime errors, which are proportional to lifetimes:

$$\delta t_{\text{life}} \approx \frac{\delta Sb}{Sb} t_{\text{life}}.$$

Numerous researchers have shown that standard deviations of relative errors of the braking parameters are 10–15% at an initial time point and within the prediction range. This error level has remained invariant over the past 30 years. Therefore, when calculating the lifetime per day, the standard deviation of determination of the fall time is usually equal to 2–3 h. When predicting for a one revolution, the corresponding error is usually found in the range ±15 min. The errors can be higher sometimes.

The problem of an increase in the prediction accuracy of satellite motion is urgent not only for determination of their lifetime, but for some other problems, such as the forecast of satellite collisions, cataloging of fine space debris, navigation, and so on. *The use of the same satellite motion model as in determination of the initial conditions from measurements* is a compulsory requirement for error minimization. This condition is not fulfilled in most cases, since the problems of forecasting and determination of initial conditions are solved *at different organizations*. To satisfy the above condition, some researchers have developed their own techniques and programs to determine the initial conditions from available measurements, which can be represented by TLE.

To determine the initial conditions from measurements, the least square technique (LST) is commonly used. This technique was developed 200 years ago, when artificial satellites did not exist. The motion of an artificial satellite is strongly affected by distorting

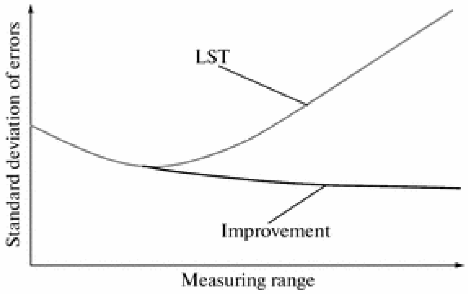

Fig. 1. Accuracy of LST estimates as a function of measurement range.

factors, which cannot be described mathematically with the necessary accuracy. Atmospheric braking is a typical example of such a disturbance; its value is proportional to a product of the real ballistic coefficient and the atmospheric density. These factors change in time unpredictably, which is why it is very difficult to take them into account during prediction. When using the LST, the effect of disturbing factors is shown in the choice of the optimal (measuring) range, i.e., a time interval, during which the measurements used are carried out. Figure 1 shows schematically the dependence of the LST estimate errors on the measuring range. The studies have shown that the optimal value depends not only on the braking parameter, but also on the accuracy and the number of measurements. In practice, this range is usually found experimentally and fixed for specific types of satellites.

The braking significantly changes while satellites are falling; therefore, the optimal measuring range also changes. In most cases, it is impossible to take into account this change during processing the LST measurements in real conditions.

Thus, the present level of errors in determining the lifetime of satellites is caused by unpredictable changes in the braking within the measuring range and while forecasting, as well as the impossibility of accurately accounting for these changes in the method of least square.

1. IMPROVEMENT OF THE TECHNIQUE FOR DETERMINATION OF INITIAL CONDITIONS AND MOTION PREDICTION

The foundation of the improved technique was published by the author almost 40 years ago (Nazarenko and Markova, 1973). In the 1970s, this technique was implemented at the Russian Space Center for the determination and prediction of orbits of LEO satellites (Nazarenko, 1991). Later, this technique was improved (Nazarenko, 1998; 2007; 2010). A specific feature of this technique is the possibility of accounting for statistical characteristics of atmospheric disturbances during measurement processing and motion prediction. This is shown in significantly different (as compared to LST) behavior of residuals between the measured and refined orbit parameters in the measuring range (table). The example relates to the processing of LST results for a rocket separated from the *Phobos-Grunt* satellite during launch (international no. 11065B).

It is seen from the table that the residual discrepancies change significantly in the measuring range when using the optimum filtering method (OFM). The main effect of the OFM consists in an increase in the accuracy of orbit determination at the last point of the measuring range, i.e., when obtaining the initial conditions for the prediction. In this case, the reduction of residuals is almost five-fold. An estimate of 0.081 s corresponds to expected errors in the initial TLE (about 500 m along the orbit). A physical sense of this effect consists in the fact that the initial measurement information does not "spread" uniformly, but is concentrated near the last point of the measurement range.

The accuracy profile of the OFM has been published (Nazarenko, 2009; 2010) in a quite general form. Three approaches to the estimation of the status vector are considered, which differ in the method of accounting for nuisance parameters (e.g., braking in the atmosphere).

1. Without accounting for nuisance parameters. The effect of nuisance parameters is not considered during the estimation of the status vector.

2. Parameterization. A vector of nuisance parameters is introduced in an extended status vector, and then the LST is applied.

3. Without parameterization (optimal measurement filtration). An a priori correlation matrix of nuisance parameters is used for "weighting" measurements

Standard deviations in residual time discrepancies (s) when using the LST and OFM

Method	Number of measurements on the dimensional range						
	k-6	k-5	k-4	k-3	k-2	k-1	k
LST	–	–	0.315	0.712	0.669	0.789	**0.394**
OFM	18.749	14.785	11.460	7.799	5.534	1.751	**0.081**