Accuracy of orbit determination and prediction for SOs in LEO. Dependence of estimate errors from accuracy and number of measurements

Andrey I. Nazarenko
Scientific Technological Center of Space Monitoring of the Earth

Abstract. The insufficient accuracy of SO motion prediction is the key problem in solution of such topical tasks, as determination of dangerous approaches and time of SO reentry, as well as cataloguing of small space debris. The investigation of accuracy for along track parameters of orbit determination for LEO space objects (SOs) is performed on the developed model. The obtained results testify to a very strong effect of system's noise level and processing interval on the accuracy of estimates of a state vector. The estimates of orbit prediction errors as function of the basic influencing factors are submitted. These estimates rather well correlate with the experience of determination of drag characteristics for various SOs.

1. Comparison of various approaches to SO state vector estimation

Now we consider the problem of estimating the SO state vector x (n×1) from measurements Z (k×1) in the classical formulation. We shall take into account the possibility of existence of some nuisance (disturbing) parameters q (m×1). In this case the basic initial relation is as follows:

$$Z = X \cdot x + B \cdot q + V \; . \tag{1}$$

Here X (k×n) and B (k×m) are known matrices, V (k×1) is the vector of measurement errors, which are accepted to be of equal accuracy and statistically independent, i. e.

$$M\left(V \cdot V^T\right) = \sigma_z^2 \cdot E \, . \tag{2}$$

The correlation matrix $M\left(q \cdot q^T\right) = \sigma_q^2 \cdot K_q$ of nuisance parameters is supposed to be known. We shall consider three approaches to state vector estimation, which differ in the technique of accounting for nuisance parameters:

I. *Without accounting for nuisance (disturbing) parameters.* In the process of state vector estimation the influence of these parameters is not taken into account. In this case the classical least-square technique (LST) is applied for estimation:

$$\hat{x} = \left(X^T \cdot X\right)^{-1} \cdot X^T \cdot Z \, . \tag{3}$$

It can easily be shown that the correlation matrix of state vector errors x is expressed as follows:

$$K_x = \sigma_z^2 \cdot \left(X^T \cdot X\right)^{-1} + \left(X^T \cdot X\right)^{-1} \cdot X^T \cdot \left(B \cdot K_q \cdot B^T\right) \cdot X \cdot \left(X^T \cdot X\right)^{-1} . \tag{4}$$

II. *Parameterization.* The state vector of nuisance (disturbing) parameters is introduced in the structure of an *extended* state vector $y^T = \left\| x \quad q \right\|^T$, and then the LST is applied. In this case the required estimate and its correlation matrix are expressed as follows:

$$\hat{y} = \begin{Vmatrix} \hat{x} \\ \hat{q} \end{Vmatrix} = \left(\begin{Vmatrix} X^T \\ B^T \end{Vmatrix} \cdot \begin{Vmatrix} X & B \end{Vmatrix} \right)^{-1} \cdot \begin{Vmatrix} X^T \\ B^T \end{Vmatrix} \cdot Z \, , \tag{5}$$

$$K_y = \begin{Vmatrix} K_{11} & K_{12} \\ K_{12}^T & K_{22} \end{Vmatrix} = \sigma_z^2 \cdot \left(\begin{Vmatrix} X^T \\ B^T \end{Vmatrix} \cdot \begin{Vmatrix} X & B \end{Vmatrix} \right)^{-1} = \sigma_z^2 \cdot \left(\begin{Vmatrix} X^T \cdot X & X^T \cdot B \\ B^T \cdot X & B^T \cdot B \end{Vmatrix} \right)^{-1} . \tag{6}$$