Construction statistical characteristics of objects sizing large then 1 mm

Consider how this problem is solved in the modern space debris models.
In the MASTER model, the determinate approach is applied in its "purest" form. In the ORDEM model, a hybrid, determinate and stochastic approach is employed.
The next step toward reducing the role of the determinate approach was made in the SDPA model: instead of the elements of orbits for each of the objects, this model uses, as the input data, the normalized statistical distributions of three elements of the orbits: the perigee altitude, eccentricity, and inclination.

Let us consider the basic principles of this technique being the most suitable for analysing the non-detectable space debris (SD) populations. We shall use only two arguments of spatial density: altitude h and geographic latitude φ. The independence of density on the longitude follows from a virtually uniform distribution of SD nodes' longitude within the altitude range under consideration. We introduce into consideration the statistic density of SD distribution over altitude and latitude:

$$p(h,\varphi) = \frac{\partial^2 N(h,\varphi)}{\partial h \partial \varphi} \quad . \tag{1}$$

Here $N(h,\varphi)$ is the number of SOs with current values of altitude h' and latitude φ', which are inside the region: $h' < h$, $\varphi' < \varphi$. Under the assumption, that the longitudinal distribution of SD is uniform, the knowledge of distribution $p(h,\varphi)$ allows easily to determine the spatial density of SOs number :

$$\rho(h,\varphi) = \frac{p(h,\varphi)}{2\pi \cdot (h+R)^2 \cdot \cos\varphi} \quad . \tag{2}$$

Here R is the Earth radius.
In constructing the spatial distribution the following data are used as the initial information:

1. N_Σ - the total amount of SOs with perigee height h_p lower than some specified value h_{max}.
2. $p(h_p)$, $p(e)$, $p(i)$ - the densities of distribution of values of perigee height, eccentricity (e) and inclination (i) of space objects in the region under consideration. For the above-mentioned densities, which are supposed to be independent, the following relations are valid:

$$\int_{h_p} p(h_p) \cdot dh_p = 1; \quad \int_e p(e) \cdot de = 1; \quad \int_i p(i) \cdot di = 1 \,. \tag{3}$$

The function $p(h,\varphi)$ is constructed in two stages. Let us consider the spherical layer with altitude boundaries $(h, h+\Delta h)$. First, we determine the average number of objects $\Delta N(h,h+\Delta h)$, which are inside the mentioned spherical layer at some fixed moment of time. At the second stage we construct the distribution $\Delta N(h,h+\Delta h,\varphi)$ of objects in the given layer over the latitude. This distribution satisfies the relationship

$$\int_\varphi \Delta N(h,h+\Delta h,\varphi) \cdot d\varphi = \Delta N(h,h+\Delta h). \tag{4}$$

Taking it into account, we obtain

$$p(h,\varphi)\cdot\Delta h = N_\Sigma\cdot\frac{\cos\varphi}{\pi}\cdot F(\varphi)\cdot\int\limits_{h_p}\int\limits_{e}\Delta\tau(h_p,e,h)\cdot\Phi(h_p,e,h)\cdot p(h_p)\cdot p(e)\cdot dh_p\cdot de\,. \qquad (5)$$

Here: $$F(\varphi)=\int\limits_{i}\frac{p(i)\cdot di}{\sqrt{\sin^2 i-\sin^2\varphi}}\text{, for } \sin i\ge\sin\varphi\,. \qquad (6)$$

Quantity $\Delta\tau(h_p, e, h)$ has a meaning of probability of finding SOs with orbital elements h_p, e, h in the spherical layer under consideration. It is determined using the technique presented in the paper (Nazarenko, 1993). The function $\Phi(h_p, e, h)$ is the following:

$$\Phi(h_p,e,h)=\frac{(1-e)^2}{\sqrt{1-e^2}}\cdot\left(\frac{h+R}{h_p+R}\right)^2. \qquad (7)$$

In the relation (5) the integrals with respect to h_p and e arguments are taken throughout the region of their possible values. In accordance with dependence (2), based on the function $p(h,\varphi)$ one can easily calculate the unknown function $\rho(h,\varphi)$, that characterises the altitude and latitude dependence of a number of SOs per unite of volume.

In accordance with the above technique, the author has developed the software for construction of SO spatial density as a function of altitude and latitude in the near-Earth space. The $N_\Sigma \cdot p(h_p)$, $p(e)$, $p(i)$ distributions are used as initial data, which are implemented as histograms and immersed into the initial data files. The program allows constructing a spatial distribution for any SOs: both catalogued and non-catalogued ones. All initial data written down into the above-mentioned files is managed by the user: the initial data can be easily corrected or fully substituted by the user either manually or by means of individual user's programs.

Examines objects larger than 1 mm. It’s possible values are divided into 8 ranges:

Range №	1	2	3	4	5	6	7	8
size, cm	0.1-0.25	0.25-0.5	0.5-1.0	1.0-2.5	2.5-5.0	5.0-10	10-20	>20

Characteristics are determined, which are used to calculate the probability and consequences of collisions satellites with space debris, namely:

- the spatial distribution of SD concentration;
- the statistical distribution of SD tangential velocity components;
- the statistical distribution of SD radial velocity components.

All these characteristics are determined for each of SD size ranges. Recall that the physical meaning of spatial density is the number of objects in volume unit (1/km³). The statistical approach is applied for solving this task because SD less than 10-20 cm in size are not catalogued.

Consider also the basis of the technique for constructing statistical distributions of tangential and radial velocity components. This is being done at the same time with construction of function (2) for spatial density. The specific values of tangential ($V_\tau(h_p,e)$) and radial ($V_r(h_p,e)$) velocity components, as well as the specific probability

$$P(h,h_p,e)=\Delta\tau(h_p,e)\cdot p(h_p)\cdot p(e)\cdot\Delta h_p\cdot\Delta e \qquad (8)$$

are correspond to some arbitrary spherical layer *(h,h+Δh)* and specific orbital parameters h_p and ***e*** from range values (h_p, $h_p + \Delta h_p$) and (*e,e+Δe*). The using of probability (8) allows us to construct the considered distributions of velocity components simple enough.

The initial distributions $p(h_p)$, *p(e)* and *p(i)* are read from files ”phd_2009.dat”, “pen.dat” and “pincl2.dat”. The first distribution is not normalized. Files are related to 2009. They prepared by the author of SDPA model on the basis of modeling the process SD pollution throughout the previous time interval. The first two distributions are constructed for each of the SD size ranges. Distributions of inclinations *p(i)* are common to all SD size ranges, but different for altitude ranges (<800, 800-1300, > 1300 km).

The calculation results for SD size ranges sizes *jd* =1, 2, ..., 8 are recorded in the files consistently:

Plotnhb.da9 – values of spatial density;
pVT.da9 - distributions of SD tangential velocity components;
pVR.da9 - distribution of SD radial velocity components.

The parts of mentioned files for first SD size range (jd = 1) are presented below.

File «Plotnhb.da9»

450	0.042	0.047	0.034	0.036	0.038	0.042	0.039	0.038	0.040	0.044	0.047	0.054	0.068	0.076	0.089	0.096	0.219	0.019
550	0.074	0.081	0.060	0.063	0.066	0.073	0.068	0.067	0.069	0.077	0.082	0.094	0.119	0.134	0.156	0.168	0.384	0.033
650	0.108	0.120	0.089	0.093	0.097	0.108	0.100	0.098	0.102	0.114	0.121	0.138	0.175	0.196	0.230	0.247	0.564	0.049
750	0.157	0.174	0.129	0.135	0.142	0.157	0.145	0.142	0.148	0.165	0.176	0.200	0.254	0.285	0.334	0.359	0.820	0.071
850	0.178	0.186	0.152	0.158	0.163	0.176	0.171	0.174	0.184	0.202	0.220	0.251	0.314	0.384	0.415	0.475	**1.000**	0.108
950	0.156	0.160	0.134	0.139	0.143	0.154	0.151	0.155	0.164	0.180	0.197	0.225	0.280	0.348	0.370	0.429	0.890	0.100
1050	0.121	0.124	0.104	0.108	0.111	0.120	0.117	0.121	0.128	0.140	0.154	0.176	0.219	0.273	0.289	0.336	0.694	0.079
1150	0.091	0.094	0.078	0.081	0.084	0.090	0.088	0.090	0.095	0.104	0.114	0.130	0.162	0.200	0.214	0.247	0.515	0.056
1250	0.080	0.083	0.068	0.071	0.073	0.079	0.076	0.078	0.083	0.091	0.099	0.113	0.141	0.174	0.186	0.214	0.449	0.049
1350	0.078	0.080	0.069	0.071	0.073	0.078	0.077	0.079	0.084	0.093	0.103	0.115	0.144	0.174	0.209	0.217	0.396	0.043
1450	0.088	0.087	0.080	0.082	0.085	0.089	0.090	0.094	0.101	0.112	0.129	0.139	0.173	0.205	0.283	0.260	0.366	0.042
1550	0.063	0.063	0.057	0.059	0.060	0.064	0.064	0.067	0.071	0.079	0.091	0.098	0.122	0.143	0.200	0.182	0.259	0.029
1650	0.048	0.048	0.042	0.044	0.045	0.048	0.047	0.049	0.052	0.058	0.066	0.072	0.090	0.105	0.143	0.133	0.204	0.022
1750	0.036	0.037	0.032	0.033	0.034	0.036	0.036	0.037	0.039	0.043	0.049	0.053	0.067	0.078	0.102	0.099	0.167	0.017
1850	0.035	0.036	0.031	0.032	0.033	0.035	0.035	0.036	0.038	0.042	0.047	0.052	0.065	0.076	0.099	0.096	0.163	0.017
1950	0.036	0.036	0.031	0.032	0.033	0.036	0.035	0.036	0.039	0.043	0.048	0.053	0.066	0.077	0.103	0.097	0.156	0.016

Spatial density maximum equals **1.037E-3** 1/km^3

Explanations. Altitude values are written in first column; normalized values of spatial density with altitude step of 100 km and latitude step of 5 degree are written into other columns.

File “pVT.da9”

450	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.024	0.529	0.245	0.056	0.084	0.000	0.000	0.000	0.052	0.010
550	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.302	0.420	0.137	0.043	0.029	0.000	0.007	0.038	0.018	0.005
650	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.159	0.434	0.229	0.059	0.048	0.000	0.005	0.019	0.043	0.000	0.005
750	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.405	0.281	0.160	0.055	0.016	0.002	0.016	0.044	0.007	0.000	0.005
850	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.093	0.558	0.188	0.054	0.029	0.001	0.011	0.031	0.029	0.000	0.000	0.005
950	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.084	0.435	0.252	0.095	0.047	0.004	0.015	0.023	0.035	0.005	0.000	0.000	0.005
1050	0.000	0.000	0.000	0.000	0.000	0.000	0.056	0.283	0.360	0.130	0.063	0.015	0.017	0.031	0.033	0.006	0.000	0.000	0.001	0.005
1150	0.000	0.000	0.000	0.000	0.000	0.007	0.195	0.396	0.203	0.068	0.023	0.016	0.033	0.037	0.012	0.002	0.000	0.001	0.002	0.004
1250	0.000	0.000	0.000	0.000	0.000	0.138	0.330	0.308	0.077	0.024	0.032	0.039	0.028	0.012	0.005	0.000	0.001	0.002	0.003	0.001
1350	0.000	0.000	0.000	0.000	0.065	0.298	0.300	0.163	0.042	0.035	0.035	0.031	0.010	0.011	0.004	0.001	0.003	0.002	0.001	0.001
1450	0.000	0.000	0.000	0.004	0.187	0.189	0.340	0.106	0.056	0.054	0.020	0.006	0.020	0.011	0.001	0.002	0.002	0.000	0.001	0.001
1550	0.000	0.000	0.000	0.051	0.239	0.311	0.177	0.068	0.075	0.025	0.009	0.020	0.016	0.001	0.003	0.002	0.001	0.001	0.002	0.000
1650	0.000	0.000	0.063	0.161	0.255	0.220	0.112	0.081	0.051	0.009	0.016	0.020	0.004	0.002	0.002	0.001	0.001	0.001	0.000	0.000
1750	0.000	0.041	0.189	0.187	0.213	0.126	0.104	0.064	0.020	0.017	0.021	0.006	0.004	0.003	0.001	0.001	0.002	0.000	0.000	0.000
1850	0.000	0.129	0.216	0.266	0.129	0.116	0.071	0.020	0.012	0.026	0.004	0.005	0.003	0.001	0.001	0.002	0.000	0.000	0.000	0.000
1950	0.000	0.305	0.158	0.278	0.109	0.077	0.025	0.008	0.026	0.004	0.006	0.003	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000

Explanations. Altitude values are written in first column. Other columns are consist of probability of finding velocity values way into ranges from 6.5+0.1·(j-1) to 6.5+0.1·j km/sec, where *j* =1, ...,20 is number of the column. The sum of the probabilities of each row is equals to 1. This distribution is presented on the figure in this section of our site.

File ” pVR.da9”

450	0.559	0.222	0.097	0.025	0.045	0.032	0.015	0.000	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
550	0.444	0.232	0.118	0.023	0.027	0.052	0.030	0.015	0.016	0.014	0.010	0.003	0.003	0.005	0.002	0.007	0.000	0.000	0.000	0.000
650	0.437	0.153	0.146	0.023	0.035	0.021	0.074	0.039	0.016	0.003	0.009	0.007	0.013	0.006	0.002	0.004	0.002	0.004	0.002	0.004
750	0.431	0.150	0.122	0.046	0.028	0.045	0.040	0.063	0.011	0.003	0.008	0.006	0.013	0.012	0.002	0.002	0.004	0.003	0.000	0.010
850	0.359	0.152	0.139	0.037	0.049	0.034	0.061	0.082	0.000	0.009	0.011	0.010	0.014	0.010	0.009	0.001	0.000	0.003	0.003	0.015
950	0.213	0.154	0.170	0.037	0.046	0.048	0.098	0.104	0.004	0.005	0.014	0.018	0.017	0.014	0.019	0.002	0.012	0.002	0.002	0.023
1050	0.162	0.139	0.128	0.023	0.055	0.052	0.132	0.110	0.011	0.009	0.017	0.022	0.025	0.014	0.036	0.005	0.002	0.011	0.003	0.047
1150	0.115	0.081	0.103	0.034	0.067	0.023	0.165	0.149	0.006	0.005	0.008	0.014	0.030	0.040	0.065	0.003	0.003	0.006	0.002	0.080
1250	0.112	0.089	0.092	0.044	0.072	0.042	0.151	0.112	0.001	0.005	0.006	0.005	0.022	0.059	0.081	0.002	0.003	0.002	0.002	0.097
1350	0.144	0.136	0.090	0.048	0.074	0.084	0.114	0.030	0.003	0.002	0.003	0.012	0.023	0.042	0.096	0.000	0.001	0.001	0.001	0.096
1450	0.267	0.122	0.100	0.115	0.037	0.067	0.036	0.017	0.005	0.011	0.012	0.009	0.021	0.039	0.063	0.000	0.001	0.001	0.001	0.077
1550	0.130	0.092	0.150	0.054	0.072	0.047	0.065	0.036	0.016	0.013	0.021	0.027	0.032	0.080	0.052	0.003	0.001	0.003	0.001	0.105
1650	0.130	0.093	0.088	0.007	0.029	0.021	0.117	0.041	0.030	0.036	0.041	0.027	0.067	0.070	0.040	0.002	0.014	0.005	0.003	0.141
1750	0.065	0.057	0.041	0.037	0.003	0.029	0.154	0.046	0.027	0.030	0.051	0.099	0.070	0.073	0.008	0.001	0.001	0.005	0.009	0.193
1850	0.060	0.047	0.068	0.029	0.029	0.043	0.155	0.064	0.051	0.014	0.041	0.082	0.048	0.061	0.000	0.000	0.001	0.001	0.001	0.205
1950	0.032	0.005	0.066	0.097	0.106	0.115	0.118	0.013	0.075	0.038	0.026	0.032	0.018	0.061	0.000	0.000	0.000	0.000	0.000	0.199

Explanations. Altitude values are written in first column. Other columns are consist of probability of finding velocity values way into ranges from 0.04·(j-1) to 0.04·j km/sec, where *j* =1, ...,20 is number of the column. The sum of the probabilities of each row is equals to 1.

The considered technique is implemented in software “plotn-09.exe”. When you buy the software, author supply mentioned initial data files, related to 2009. User can edit them, while maintaining unchanged form.

Литература

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